



HOMework 03

PSTAT 120B: Mathematical Statistics, I
Summer Session A, 2024 with Instructor: Ethan P. Marzban

1. **(Modified from #6.95)** Let $Y_1, Y_2 \stackrel{\text{i.i.d.}}{\sim} \text{Unif}[0, 1]$.
- Find the density of $U_1 := -\ln(Y_1 Y_2)$. Use this to recognize the distribution of U_1 by name, including any/all relevant parameter(s)!
 - Find the density of $U_2 := Y_1 Y_2$
 - Find the density of $U_3 := Y_1^2$. Is this the same as the density of U_2 from part (b) above?

2. Let $Y_1, Y_2 \stackrel{\text{i.i.d.}}{\sim} \text{Exp}(\theta)$. Find the distribution of U , where

$$U := \frac{Y_2}{Y_1 + Y_2}$$

Be sure to include both the distribution's name as well as any/all relevant parameter(s)!

3. In this problem, we'll consider the exercise of deriving the distribution of the minimum of two *non*-i.i.d. random variables. Suppose $(X, Y) \sim f_{X,Y}$, where

$$f_{X,Y} = 2e^{-(x+y)} \cdot \mathbb{1}_{\{0 \leq x \leq y < \infty\}}$$

Define $U := \min\{X, Y\}$ to be the minimum of X and Y .

- Argue that $\overline{F}_U(u)$, the survival of U , is given by $\mathbb{P}(X > u, Y > u)$. [Yes, we've done this before but it's good practice to revisit the argument for why this fact holds!]
- Compute $\overline{F}_U(u)$ as a function of u . **Hint:** You should sketch the region of integration here.
- Use your answer to part (b) to identify the distribution of U by name, including any/all relevant parameter(s).

4. Let $Y_1, Y_2, \dots \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(0, 1)$.

- What is the distribution of $U_1 := \sum_{i=1}^6 Y_i^2$? Include both the distribution's name along with any relevant parameter(s)!
- What is the distribution of $U_2 := \sum_{i=1}^6 (Y_i - \bar{Y}_6)^2 + Y_7^2$, where $\bar{Y}_6 := (1/6) \sum_{i=1}^6 Y_i$? Include both the distribution's name along with any relevant parameter(s)!
- What is the distribution of $U_3 := 4\bar{Y}_{16}/S_{16}$, where

$$\bar{Y}_{16} := \frac{1}{16} \sum_{i=1}^{16} Y_i \quad \text{and} \quad S_{16} := \sqrt{\frac{1}{15} \sum_{i=1}^{16} (Y_i - \bar{Y}_{16})^2}$$

Include both the distribution's name along with any relevant parameter(s)!