Selected Discrete Distributions

Distribution	PMF	Expec./Var.	MGF
$X \sim \text{Bin}(n, p)$	$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$, $k \in \{0, \cdots, n\}$	$\mathbb{E}[X] = np$ $\text{Var}(X) = np(1-p)$	$M_X(t) = (1 - p + pe^t)^n$
$X \sim \text{Geom}(p)$	$p_X(k) = p(1-p)^{k-1}$, $k \in \{1, 2, \cdots\}$	$\mathbb{E}[X] = \frac{1}{p}$ $\text{Var}(X) = \frac{1-p}{p^2}$	$M_X(t) = \frac{pe^t}{1-(1-p)e^t}$, $t < -\ln(1-p)$
$X \sim \text{Geom}(p)$	$p_X(k) = p(1-p)^k$, $k \in \{0, 1, \cdots\}$	$\mathbb{E}[X] = \frac{1-p}{p}$ $\text{Var}(X) = \frac{1-p}{p^2}$	$M_X(t) = \frac{p}{1-(1-p)e^t}$, $t < -\ln(1-p)$
$X \sim \text{NegBin}(r, p)$	$p_X(k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}$, $k \in \{r, r+1, \cdots\}$	$\mathbb{E}[X] = \frac{r}{p}$ $\text{Var}(X) = \frac{r(1-p)}{p^2}$	$M_X(t) = \left[\frac{pe^t}{1-(1-p)e^t}\right]^r$, $t < -\ln(1-p)$
$X \sim \text{Pois}(\lambda)$	$p_X(k) = e^{-\lambda} \cdot \frac{\lambda^k}{k!}$, $k \in \{0, 1, \cdots\}$	$\mathbb{E}[X] = \lambda$ $\text{Var}(X) = \lambda$	$M_X(t) = \exp\left\{\lambda(e^t - 1)\right\}$

Selected Continuous Distributions

Distribution	PDF	Expec./Var.	MGF
$X \sim \text{Unif}[a, b]$	$f_X(x) = \dfrac{1}{b-a} \cdot \mathbb{1}_{\{x \in [a,b]\}}$	$\mathbb{E}[X] = \frac{a+b}{2}$ $\text{Var}(X) = \frac{(b-a)^2}{12}$	$M_X(t) = \begin{cases} \frac{e^{tb}-e^{ta}}{t(b-a)} & \text{if } t \neq 0 \\ 1 & \text{if } t = 0 \end{cases}$
$X \sim \text{Exp}(\beta)$	$f_X(x) = \dfrac{1}{\beta} e^{-x/\beta} \cdot \mathbb{1}_{\{x \geq 0\}}$	$\mathbb{E}[X] = \beta$ $\text{Var}(X) = \beta^2$	$M_X(t) = (1 - \beta t)^{-1}$, $t < 1/\beta$
$X \sim \text{Gamma}(\alpha, \beta)$	$f_X(x) = \dfrac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta} \cdot \mathbb{1}_{\{x \geq 0\}}$	$\mathbb{E}[X] = \alpha\beta$ $\text{Var}(X) = \alpha\beta^2$	$M_X(t) = (1 - \beta t)^{-\alpha}$, $t < 1/\beta$
$X \sim \mathcal{N}(\mu, \sigma^2)$	$f_X(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\dfrac{(x-\mu)^2}{2\sigma^2}\right\}$	$\mathbb{E}[X] = \mu$ $\text{Var}(X) = \sigma^2$	$M_X(t) = \exp\left\{\mu t + \frac{\sigma^2}{2} t\right\}$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2} \qquad \int_y^\infty \frac{1}{\theta} e^{-x/\theta}\,\mathrm{d}x = e^{-y/\theta} \qquad \Gamma(r) := \int_0^\infty t^{r-1} e^{-t}\,\mathrm{d}t$$

- If $Y \sim \mathcal{N}(0,1)$, then $Y^2 \sim \chi^2_1$.
- If $Y \sim \mathcal{N}(\mu, \sigma^2)$, then $U := [(Y - \mu)/\sigma] \sim \mathcal{N}(0,1)$
- Given independent random variables $\{Y_i\}_{i=1}^n$ with $Y_i \sim \chi^2_{\nu_i}$, then $(Y_1 + \cdots + Y_n) \sim \chi^2_{\nu_1 + \cdots + \nu_n}$
- Given $Y_1, Y_2, \cdots \overset{\text{i.i.d.}}{\sim} \mathcal{N}(\mu, \sigma^2)$, $\overline{Y}_n := \frac{1}{n}\left(\sum_{i=1}^n Y_i\right)$ and $S_n := \sqrt{\frac{1}{n-1}\sum_{i=1}^n (Y_i - \overline{Y}_n)^2}$,

$$\sqrt{n}\left(\frac{\overline{Y}_n - \mu}{\sigma}\right) \sim \mathcal{N}(0,1); \qquad \sqrt{n}\left(\frac{\overline{Y}_n - \mu}{S_n}\right) \sim t_{n-1}; \qquad \left(\frac{n-1}{\sigma^2}\right) S_n^2 \sim \chi^2_{n-1}$$

- $f_{Y_{(1)}} = n[1 - F_Y(y)]^{n-1} f_Y(y);$ $\qquad f_{Y_{(n)}}(y) = n[F_Y(y)]^{n-1} f_Y(y)$