Selected Discrete Distributions

Distribution	PMF	Expec./Var.	MGF
$X \sim \text{Bin}(n, p)$	$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$, $k \in \{0, \cdots, n\}$	$\mathbb{E}[X] = np$ $\text{Var}(X) = np(1-p)$	$M_X(t) = (1 - p + pe^t)^n$
$X \sim \text{Geom}(p)$	$p_X(k) = p(1-p)^{k-1}$, $k \in \{1, 2, \cdots\}$	$\mathbb{E}[X] = \frac{1}{p}$ $\text{Var}(X) = \frac{1-p}{p^2}$	$M_X(t) = \frac{pe^t}{1-(1-p)e^t}$, $t < -\ln(1-p)$
$X \sim \text{Geom}(p)$	$p_X(k) = p(1-p)^k$, $k \in \{0, 1, \cdots\}$	$\mathbb{E}[X] = \frac{1-p}{p}$ $\text{Var}(X) = \frac{1-p}{p^2}$	$M_X(t) = \frac{p}{1-(1-p)e^t}$, $t < -\ln(1-p)$
$X \sim \text{NegBin}(r, p)$	$p_X(k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}$, $k \in \{r, r+1, \cdots\}$	$\mathbb{E}[X] = \frac{r}{p}$ $\text{Var}(X) = \frac{r(1-p)}{p^2}$	$M_X(t) = \left[\frac{pe^t}{1-(1-p)e^t}\right]^r$, $t < -\ln(1-p)$
$X \sim \text{Pois}(\lambda)$	$p_X(k) = e^{-\lambda} \cdot \frac{\lambda^k}{k!}$, $k \in \{0, 1, \cdots\}$	$\mathbb{E}[X] = \lambda$ $\text{Var}(X) = \lambda$	$M_X(t) = \exp\left\{\lambda(e^t - 1)\right\}$

Selected Continuous Distributions

Distribution	PDF	Expec./Var.	MGF
$X \sim \text{Unif}[a, b]$	$f_X(x) = \dfrac{1}{b-a} \cdot \mathbb{1}_{\{x \in [a,b]\}}$	$\mathbb{E}[X] = \frac{a+b}{2}$ $\text{Var}(X) = \frac{(b-a)^2}{12}$	$M_X(t) = \begin{cases} \frac{e^{tb}-e^{ta}}{t(b-a)} & \text{if } t \neq 0 \\ 1 & \text{if } t = 0 \end{cases}$
$X \sim \text{Exp}(\beta)$	$f_X(x) = \dfrac{1}{\beta} e^{-x/\beta} \cdot \mathbb{1}_{\{x \geq 0\}}$	$\mathbb{E}[X] = \beta$ $\text{Var}(X) = \beta^2$	$M_X(t) = (1 - \beta t)^{-1},\ t < 1/\beta$
$X \sim \text{Gamma}(\alpha, \beta)$	$f_X(x) = \dfrac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta} \cdot \mathbb{1}_{\{x \geq 0\}}$	$\mathbb{E}[X] = \alpha\beta$ $\text{Var}(X) = \alpha\beta^2$	$M_X(t) = (1 - \beta t)^{-\alpha},\ t < 1/\beta$
$X \sim \mathcal{N}(\mu, \sigma^2)$	$f_X(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\dfrac{(x-\mu)^2}{2\sigma^2}\right\}$	$\mathbb{E}[X] = \mu$ $\text{Var}(X) = \sigma^2$	$M_X(t) = \exp\left\{\mu t + \frac{\sigma^2}{2} t\right\}$

Calculus Results

- **Product Rule:** $\dfrac{\mathrm{d}}{\mathrm{d}x}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
- **Quotient Rule:** $\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$
- **Power Rule:** $\dfrac{\mathrm{d}}{\mathrm{d}x}(x^n) = \begin{cases} nx^{n-1} & \text{if } n \neq -1 \\ \ln(x) & \text{if } n = -1 \end{cases}$
- **Chain Rule:** $\dfrac{\mathrm{d}}{\mathrm{d}x} f(g(x)) = f'(g(x)) \cdot g'(x)$
- **Gamma Function:** $\Gamma(r) := \int_0^\infty t^{r-1} e^{-t}\, \mathrm{d}t$ if $r > 0$
- **Recursiveness of Gamma Fnt.:** $\Gamma(r) = (r-1) \cdot \Gamma(r-1)$

Selected Statistics Results

- **Standarization of Normals:** If $Y \sim \mathcal{N}(0,1)$, then $Y^2 \sim \chi^2_1$.
- **Closure of χ^2 under Sums:** Given independent random variables $\{Y_i\}_{i=1}^n$ with $Y_i \sim \chi^2_{\nu_i}$, then
$$(Y_1 + \cdots + Y_n) \sim \chi^2_{\nu_1 + \cdots + \nu_n}$$
- **Closure of Gamma under Scaling:** if $Y \sim \text{Gamma}(\alpha, \beta)$, then $(cY) \sim \text{Gamma}(\alpha, c\beta)$ for $c > 0$
- **Sample Mean:** $\overline{Y}_n := \frac{1}{n}\sum_{i=1}^n Y_i$ **Sample Variance:** $S_n^2 := \frac{1}{n-1}\sum_{i=1}^n (Y_i - \overline{Y}_n)^2$

  Sample Standard Deviation: $S_n := \sqrt{S_n^2} = \sqrt{\frac{1}{n-1}\sum_{i=1}^n (Y_i - \overline{Y}_n)^2}$
- **Modified Standardization Result:** Given $Y_1, Y_2, \cdots \overset{\text{i.i.d.}}{\sim} \mathcal{N}(\mu, \sigma^2)$,
$$\sqrt{n}\left(\frac{\overline{Y}_n - \mu}{S_n}\right) \sim t_{n-1}$$
- **Sampling Distribution of First Order Statistic:** $f_{Y_{(1)}} = n[1 - F_Y(y)]^{n-1} f_Y(y)$

  Sampling Distribution of n^{th} Order Statistic: $f_{Y_{(n)}}(y) = n[F_Y(y)]^{n-1} f_Y(y)$
- **Factorization Theorem:** $\mathcal{L}_{\vec{\boldsymbol{Y}}}(\theta) = g(U, \theta) \times h(\vec{\boldsymbol{Y}}) \iff U$ is sufficient for θ.
- **Equivariance Property of MLE:** $\widehat{\tau(\theta)}_{\text{MLE}} = \tau\left(\widehat{\theta}_{\text{MLE}}\right)$