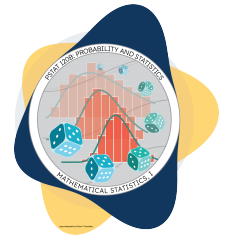


# DISCUSSION WORKSHEET 01

**PSTAT 120B:** Mathematical Statistics, I  
**Summer Session A, 2024** with Instructor: Ethan P. Marzban



Welcome to our first PSTAT 120B Discussion Section! Discussion worksheets are designed to give you additional practice with material covered in lecture.

## *Conceptual Review*

- What is a **conditional mass/density function**? For what values is it defined?
- What is a **conditional expectation**?
- What are the **Law of Iterated Expectations** and **Law of Total Variance**?
- What is the **Gamma distribution**? Specifically, what is its density function? What are its expectation and variance?

## *Problem 1: Conditional Distributions/Expectations*

Let  $(X, Y)$  be a bivariate random vector with joint probability density function (p.d.f.) given by

$$f_{X,Y}(x, y) = \begin{cases} \lambda y e^{-y(x+\lambda)} & \text{if } x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where  $\lambda > 0$  is a fixed constant.

- Find  $f_Y(y)$ , the marginal density of  $Y$  and use this to identify  $Y$  as belonging to a known distribution. **Be sure to include any/all relevant parameter(s)!**
- Find  $f_{X|Y}(x | y)$ , the density of  $(X | Y = y)$ , and use this to identify  $(X | Y = y)$  as belonging to a known distribution. **Be sure to include any/all relevant parameter(s)!**
- Set up ~~but do not evaluate~~ and integral corresponding to  $E[X]$ , that only involves the marginal density function of  $Y$ .

**Hint:** Law of Iterated Expectations

*Problem 2: The Gamma Distribution*

Recall (from lecture) that if  $X \sim \text{Gamma}(\alpha, \beta)$ , then  $X$  has density given by

$$f_X(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta} \cdot \mathbb{1}_{\{x \geq 0\}}$$

where  $\Gamma(\alpha)$  denotes the **gamma function**, defined as

$$\Gamma(r) := \int_0^\infty x^{r-1} e^{-x} dx \text{ if } r \geq 0$$

and  $\Gamma(0) := 1$ .

**a)** Show that  $X$  has MGF (moment generating function) given by

$$M_X(t) = \begin{cases} (1 - \beta t)^{-\alpha} & \text{if } t < 1/\beta \\ \infty & \text{otherwise} \end{cases}$$

**b)** Let  $Y \sim \chi^2_\nu$ . Use your answer to part (a) to derive the MGF  $M_Y(t)$  of  $Y$ .

**c)** What is another name for the  $\chi^2_2$  distribution? Be sure to give the distribution's name and also list out any/all relevant parameter(s)!