DISCUSSION WORKSHEET 05

PSTAT 120B: Mathematical Statistics, I
Summer Session A, 2024 with Instructor: Ethan P. Marzban

Conceptual Review

(a) What is a **population moment**? What is a **sample moment**?

(b) What is the **Method of Moments**? Does the Method of Moments produce consistent estimators? What about unbiased estimators?

(c) What is a **likelihood**?

Problem 1

Let $Y_1, \cdots, Y_n \overset{\text{i.i.d.}}{\sim} f(y;\alpha)$ where

$$f(y;\alpha) = \frac{1+\alpha y}{2} \cdot \mathbb{1}_{\{-1 \leq y \leq 1\}}$$

where $\alpha \in [-1, 1]$ is an unknown parameter.

(a) Find $\widehat{\alpha}_{\text{MM}}$, the method of moments estimator for α.

(b) Show that $\widehat{\alpha}_{\text{MM}}$ is a consistent estimator for α.

(c) Suppose we obtain the following observed instance of a sample from the above-stated distribution:

$$\vec{\boldsymbol{y}} = (-0.38,\ 0.61,\ -0.13,\ 0.79,\ 0.89,\ -0.90,\ 0.11,\ 0.80)$$

What is an appropriate estimate for α based on this sample, using the method of moments?

(d) What is the approximate sampling distribution of $\widehat{\alpha}_{\text{MM}}$, assuming a large sample size?

(e) Use your answer to part (d) to approximate the probability that $\widehat{\alpha}_{\text{MM}}$ exceeds 1. Why is this a problem (i.e. why would we want this probability to be as small as possible)?