DISCUSSION WORKSHEET 07

PSTAT 120B: Mathematical Statistics, I
Summer Session A, 2024 with Instructor: Ethan P. Marzban

Conceptual Review

(a) What is the **Cramér-Rao Lower Bound**? How does that relate to the **Fisher Information**?

(b) What is an **efficient estimator**?

(c) What is a **confidence interval**? How do we construct one assuming normality in the population?

(d) What is a **pivotal quantity**? How do we use these to construct confidence intervals?

Problem 1:

Let $Y_1, \cdots, Y_n \overset{\text{i.i.d.}}{\sim} \text{Unif}[0, \theta]$, where both $\theta > 0$ is an unknown parameter.

(a) Propose a pivot for θ that is a function of $Y_{(n)}$, and show that your proposed quantity is in fact a pivot.

(b) Use your pivot from part (a) to construct a $(1-\alpha) \times 100\%$ confidence interval for θ.

(c) Suppose we collect the following observed instance of a sample: $\vec{\boldsymbol{y}} = (0.74,\ 1.97,\ 0.31,\ 0.18,\ 0.28)$. Construct a 95% confidence interval for θ.