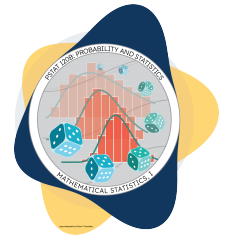


DISCUSSION WORKSHEET 08

PSTAT 120B: Mathematical Statistics, I
Summer Session A, 2024 with Instructor: Ethan P. Marzban



Conceptual Review

- What is a **hypothesis test**? How do the **null and alternative hypotheses** factor into this?
- What are the different types of alternative hypotheses?
- What is a **Type I Error**? What about a **Type II Error**?
- What is a **Z -test**? What about a **T -test**? How do you know which one to use?
- What is a **p -value**? How do you compute it?
- What is the **level of significance**? What about the **power function**? How do these relate to Type I and Type II Errors?

Problem 1 (Modified from 10.5):

Let Y_1 and Y_2 be independent and identically distributed with a uniform distribution over the interval $(\theta, \theta + 1)$. For testing $H_0 : \theta = 0$ versus $H_A : \theta > 0$, we have two competing tests:

Test 1: Reject H_0 if $Y_1 > 0.95$

Test 2: Reject H_0 if $Y_1 + Y_2 > c$

- Find the value of c that ensures Test 2 has the same level of significance as Test 1.
- Derive an expression for the power curve of Test 1, and sketch it.
- (Hard; Come back to this one at the end, if you have time)** Derive an expression for the power curve of Test 2, and sketch it. What can you conclude about the power of test 2 compared to the power of test 1 *for all* $\theta \geq 0$?

Hint: Take a look at Quiz 2, where we derived the density for the sum of two i.i.d. $\text{Unif}[0, 1]$ random variables.

Problem 2:

GachoBuzz claims that the average lifetime of one of their batteries is 5 years. To test these claims, Yosef takes an i.i.d. sample of 25 *GachoBuzz*-brand batteries, and finds that his sample has an average lifetime of 4.9 years. Assume that the lifetimes of *GachoBuzz*-brand batteries is normally distributed, with a standard deviation of 0.1 years. Suppose Yosef wants to use his data to test *GachoBuzz*'s claims against a two-sided alternative, using a 5% level of significance.

- (a) Let μ denote the average lifetime of a *GachoBuzz*-brand battery. State the null and alternative hypotheses.
- (b) Compute the observed value of the test statistic.
- (c) What is the critical value of this test? You may leave your answer in terms of an inverse-CDF, if necessary.
- (d) What is the p -value of Yosef's observed test statistic? You may leave your answer in terms of a CDF, if necessary.
- (e) Use a computer software to numerically evaluate the p -value expression you found in part (d) above. Then, conduct the test and phrase your conclusions in the context of the problem.